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B.Sc (Part-1) Hons
Paper-1

Amplitude Resonance of forced oscillation

The amplitute of the forced oscillation is

$$A = \frac{f_0}{\sqrt{[(\omega^2-p^2)^2 + 4r^2p^2}} \quad \text{——— (1)}$$

This equation is shows that the amplitude of the forced oscillation depends upon (ω^2-p^2) which in turn depends upon the difference between the driving frequency p and the natural frequency ω of the oscillator. Smaller this difference, longer the amplitude.

At very low driving frequency $(p<<\omega)$, we have

$$A \simeq \frac{f_0}{\omega^2} \simeq \frac{F_0/m}{k/m}$$

$$A \simeq \frac{F_0}{k} \quad \text{——— (2)}$$

which shows that the amplitude, which now depends upon the mass, continuously decreases as the driving frequency.

At very high driving frequency $(p>>\omega)$, we have

$$A \simeq \frac{f_0}{p^2} \simeq \frac{F_0}{mp^2} \quad \text{——— (3)}$$

which shows that the amplitude, which now depends upon the mass, continuously decreases as the driving frequency p is further increased.

The amplitude is maximum for a particular frequency. When the amplitude is maximum, the phenomenon is called the amplitude resonance and the particular driving frequency is called the "resonant frequency".

For the amplitude to be maximum, the denominator of the eqn (1) must be minimum. For this

$$\frac{d}{dp}\left[(\omega^2-p^2)^2+4r^2p^2\right]=0 \quad \text{——— (4)}$$

This gives rise to $p=\omega$

Thus, the amplitude resonance occurs when the frequency of the impressed force coincides with the natural frequency of the oscillator.

This is the condition of resonance in the absence of damping.

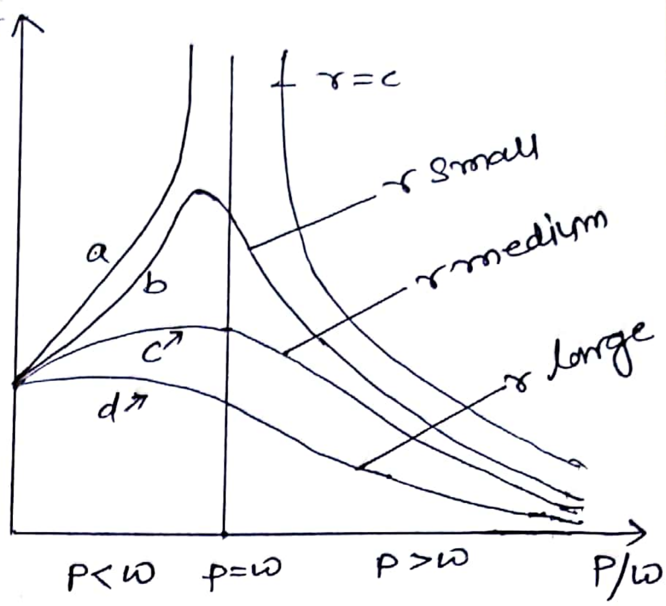
If the damping factor r is not negligible in this case the frequency of the impressed force is $p=\sqrt{\omega^2-2r^2}$ and hence from equation (1), the maximum amplitude becomes

$$A_{max}=\frac{f_0}{2r\sqrt{r^2+p^2}} \quad \text{——— (5)}$$

This shows the dependence of maximum amplitude on damping r. In ideal case for $r=0$, $A_{max}\rightarrow\infty$. Thus we see that damping controls the responce at resonance. For small damping r^2 can be neglected in eqn (5)

$$A_{max}=\frac{f_0}{2rp}=\frac{f_0}{2r\omega} \qquad (\because p=\omega)$$

In this figure the amplitude of force oscillation A has been plotted against the ratio P/ω for various dampings. For very low driving frequency p, the amplitude nearly same for all values of damping.



As p increases, the amplitude increases and becomes maximum at a certain value of p, which depends on damping. Curve (a) shows the amplitude when r=0, that is no damping. In this case the amplitude becomes infinite at p=ω. Curve (b), (c) and (d) shows that as damping (r) increases the peak of the curve moves towards the left i.e. the value of p for which the amplitude is a maximum decreases. Further, as damping (r) increases, the peak moves downwards i.e. the maximum amplitude of the forced oscillation is lowered.

As p further increases, the amplitude again falls and tends towards zero whatever be damping.

Sharpness of Resonance! — The amplitude of forced oscillation is maximum when the frequency of the applied force has a value which satisfies the condition of resonance. A soon as the frequency changes from this value, the amplitude falls. When the fall in amplitude for a small departure of the frequency from the resonant value is considerable, the resonance is said to be 'sharp'. If on the other hand, the fall is small, the resonance is said to be 'flat'.